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Influence of Discontinuous Yielding on Normal Anisotropy (*R*-Value) Measurements

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Accurate assessment of plastic anisotropy is essential to many material models used in computer-aided sheet metal forming simulations. In this analysis, the influence of discontinuous yielding (yield point elongation or YPE) on normal anisotropy (*R*-value) measurements was examined. A laboratory-annealed, non temper-rolled, aluminum-killed drawing quality (AKDQ) steel was considered. It was found that discontinuous yielding significantly influences *R*-value calculations by imposing an effective offset to the width strain measurement during a sheet tensile test if the *R*-value is defined in terms of total strains ($R = \varepsilon_W/\varepsilon_T$). The effect may be positive or negative and shows considerable variability between tests. For a more accurate and consistent representation of normal anisotropy, the incremental *R*-value ($R' = d\varepsilon_W/d\varepsilon_T$) should be used in cases of significant YPE. A procedure has been developed to account for the low-strain effects of discontinuous yielding on plastic anisotropy measurements, where the R'-value is determined in a region of stable, uniform deformation on the stress/strain curve. For adequately temper rolled materials (with minimal YPE) and for inherently YPE-free materials, the *R*-value may be determined in the conventional way.

Keywords anisotropy, automotive, mechanical testing, *R*-value, stamping, yield point elongation (YPE)

1. Introduction

Normal anisotropy, or *R*-value, is a term that has long been used to describe the relative "resistance to thinning" of sheet metal during forming and is generally linked to deep drawability in low-carbon steels, for example (Ref 1). The *R*-value is also a convenient and meaningful way to represent anisotropy in plastic deformation in a more general sense, without specific implication toward deep drawing performance. *R*-values have shown significance in yielding theories and material models since before the correlation between *R*-value and drawability was made (e.g., Hill's 1948 Yield Function, Ref 2). In some more recently developed and widely applied material models (e.g., Barlat's 1996 Yield Function, Ref 3), *R*-values are not explicit to the mathematical formulation, per se. However, experimentally determined *R*-values may be used in part to determine the explicitly defined coefficients of plastic anisotropy of a particular material model (Ref 4). Furthermore, as the demand for rapid, accurate simulation capability increases in computer-aided engineering programs, the importance of precise and valid measurements of plastic anisotropy increases correspondingly.

1.1 Definition and Measurement of R-Values

Commonly, in many engineering textbooks (Ref 5-7), the *R*-value is defined as the ratio of the true width strain to the true thickness strain in a tensile test, or:

$$R \equiv \frac{\varepsilon_W}{\varepsilon_T} \quad \text{(Eq 1)}$$

where ε_W and ε_T are the width and thickness strains, respectively. As the thickness strain is difficult to measure with precision during a tensile test of a thin sheet specimen, the plastic deformation volume constancy assumption is typically used, where:

$$\varepsilon_L + \varepsilon_W + \varepsilon_T = 0 \rightarrow \varepsilon_T = -(\varepsilon_L + \varepsilon_W) \rightarrow R = \frac{\varepsilon_W}{-(\varepsilon_L + \varepsilon_W)} \quad \text{(Eq 2)}$$

where ε_L is the length strain. Thus, only the width strain and the length strain are measured while the thickness strain is calculated.[1] Whenever an *R*-value is reported, the strain at which the parameter was determined should be included (with a subscript, for example). Commonly reported *R*-values are R_{10} and R_{15} (measured at 10 and 15% elongation, respectively).

An alternate and more appropriate definition of the *R*-value is the ratio between the incremental width and thickness strains in a sheet tensile test (Ref 8). With this distinction, the incremental *R*-value, R', may be defined as:

$$R' \equiv \frac{d\varepsilon_W}{d\varepsilon_T} \quad \text{(Eq 3)}$$

where $d\varepsilon_W$ and $d\varepsilon_T$ are the incremental width and thickness strains. The *R*-value is usually defined in terms of total strains (Eq 1) for convenience and because, in many cases, the *R*-value does not vary appreciably with strain (Ref 6). The incremental

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[1] Strictly speaking, all strain values are calculated values based on displacement measurements; however, in the present context, "measured" strains are those that are calculated directly from displacement measurements rather than from the volume constancy assumption.

R-value, R', may be described in terms of the slope of the ε_W versus ε_L curve determined by a tensile test with axial and transverse extensometers as follows:

$$R' = \frac{d\varepsilon_W}{d\varepsilon_T} = \frac{d\varepsilon_W/d\varepsilon_L}{d\varepsilon_T/d\varepsilon_L} = \frac{d\varepsilon_W/d\varepsilon_L}{-d(\varepsilon_L + \varepsilon_W)/d\varepsilon_L}$$
$$= \frac{d\varepsilon_W/d\varepsilon_L}{-(1 + d\varepsilon_W/d\varepsilon_L)} = -\left(\frac{\lambda'}{1+\lambda'}\right) \quad \text{(Eq 4)}$$

where $\lambda' = d\varepsilon_W/d\varepsilon_L$ within the uniform deformation portion of the stress/strain curve. In Eq 4, λ' is used to distinguish from $\lambda = \varepsilon_W/\varepsilon_L$(defined in Ref 9) where it is assumed that λ' is constant within the measured strain interval (a reasonable assumption in most cases).

1.2 Rationale and Objectives for Current Analysis

For many engineering alloys, yield point elongation (YPE) is negligible either because the material has been sufficiently temper rolled to remove YPE or because the material is inherently free of YPE (e.g., fully stabilized steels, dual phase steels, and various aluminum alloys). As an example, for exposed automotive steel body panels where surface appearance is critical, it is crucial that YPE be minimized to avoid the formation of strain lines or Lüders bands during stamping. In many other cases, however, commercially produced materials exhibit various degrees of YPE (structural or nonexposed components where surface appearance is not critical). Such examples include hot-rolled, batch annealed, continuously annealed, and galvanized low-carbon and high-strength low-alloy (HSLA) steels.

Additionally, in certain developmental laboratory studies, it is not uncommon to evaluate materials in the annealed (non-temper-rolled) condition, where otherwise the commercial strip material would be temper-rolled in production. Examples include batch annealing, continuous annealing, and galvanizing simulations where sample size is oftentimes small, and laboratory temper rolling is logistically difficult or impossible. It is foreseeable that pronounced discontinuous yielding affects R-value measurements when the percent YPE is a significant fraction of the R-value calculation strain (i.e., X in R_X). In fact, the ASTM Standard (E 517) for R-value determination states that the accuracy and reproducibility of R-value calculations will be reduced unless the test is continued beyond the YPE (Ref 10). The objectives of this analysis are to illustrate the influence of discontinuous yielding on R-value calculations and to establish a method by which normal anisotropy may be evaluated for cases of significant YPE.

Table 1 Laboratory batch annealing cycle parameters

Heating rate, °C/h (°F/h)	Soak temperature, °C (°F)	Soak time, h	Atmosphere, H_2/N_2
28 (50)	720 (1300)	10	15/85

Table 2 Nominal composition of AKDQ steel (wt.%)

C	Mn	P	S	Si	Al	N	Fe
0.04	0.30	0.010	0.010	0.015	0.045	0.005	Balance

2. Experimental Procedures

2.1 Material Preparation

As part of a separate investigation, a 1.1 mm (0.043 in.) thick cold-rolled (full hard), aluminum-killed drawing quality (AKDQ) sheet steel was batch annealed in a laboratory tube furnace. Due to laboratory annealing furnace dimensional limitations, 250 × 25 mm (10 × 1 in.) rectangular tensile specimen blanks were cut to size prior to the batch annealing simulation, so it was not possible to temper roll the material after the annealing simulation (except, perhaps, in the original rolling direction). The batch annealing simulation parameters are given in Table 1, and the nominal steel composition is given in Table 2. The AKDQ steel was produced through cold rolling at the United States Steel Corporation Mon Valley Works (Dravosburg, PA), and the batch annealing simulation parameters were selected, from experience, to induce complete recrystallization and the beneficially high R-values (>1) normally associated with drawing quality steels.

2.2 Tensile Testing

Tensile tests were run according to ASTM Standard E 517 (Ref 10) at a constant actuator displacement rate of 2.5 mm/min (0.1 in:min). During each tensile test, the axial (longitudinal) and transverse (width) strains were measured with standard extensometers and digitally recorded. Four tensile tests were run in each of three orientations: 0°, 45°, and 90° to the sheet rolling direction [longitudinal (L), diagonal (D) and transverse (T) directions, respectively].

3. Results

3.1 Tensile Properties

The average tensile properties for each orientation are listed in Table 3. Minor variations in strength and ductility parameters are evident between orientations, and the most significant variation is in the total elongation (TE) value (40-48%). Note that the upper yield strength (UYS) is higher than the ultimate tensile strength (UTS) and that the YPE is between 4 and 6% in all orientations, both characteristic of well-annealed, non-temper-rolled drawing quality steel. Examples of engineering stress/strain curves in each orientation are shown in Fig. 1(a), and the yielding behavior is magnified in Fig. 1(b).

3.2 Normal Anisotropy (R-Values)

The R-values were determined according to Eq 1 (typical industrial practice) at 10 and 15% elongation (R_{10} and R_{15}, respectively). The R-values are reported for each of the four tests in each orientation in Table 4. Significant variation in

Table 3 Directional tensile properties

Orientation	UYS, MPa	LYS, MPa	YPE, %	UTS, MPa	UE, %	TE, %
L	344	208	5.5	302	27.3	40.3
D	344	223	5.2	302	28.2	47.9
T	359	212	4.5	301	28.0	46.3

Average of four tests/orientation: UYS (LYS) = upper (lower) yield strength, YPE = yield point elongation, UTS = ultimate tensile strength, UE (TE) = uniform (total) elongation

R-value between specimens in a given orientation is apparent. For example, the R_{10} values in the L direction range from 0.98 to 1.44, and from 1.32 to 1.94 in the T direction. As a check, the mean normal anisotropy (R_m) and the planar anisotropy (ΔR) were estimated by the Modul-R method. The R_m-value and the ΔR-value are defined as:

$$R_m \equiv \tfrac{1}{4}(R_0 + 2R_{45} + R_{90}),\ \text{and}\ \Delta R \equiv \tfrac{1}{2}(R_0 - 2R_{45} + R_{90}) \quad \text{(Eq 5)}$$

where the subscripts 0, 45, and 90 refer to the orientation (degrees) with respect to the sheet rolling direction (i.e., L, D, and T directions). With the Modul-R method, the resonant frequencies of the test material are determined in the L, D, and T directions by a magnetostrictive oscillator. There exists a fundamental relationship between the resonant frequency and the Young's modulus E in each orientation. Empirical relationships for low-carbon steels have been established between E_m and R_m, and between ΔE and ΔR, where the expressions for E_m and ΔE are identical in form to those given in Eq 5 for R_m and ΔR (Ref 11). The correlation arises from the concurrent dependence of elastic stiffness and plastic anisotropy on crystallographic texture. The Modul-R method estimates an R_m-value of 1.44 and a ΔR-value of 0.39 for the AKDQ steel in this analysis. The R_m and ΔR-values were calculated based upon the average R_{10} and R_{15} values given in Table 4 for each orientation, and the results are shown in contrast to the values determined by the Modul-R method in Fig. 2, and listed in Table 5.

Table 4 Directional anisotropy parameters

Orientation(a)	Test No.	R_{10} value	R_{15} value
L (RD)	1	1.20	1.24
	2	0.98	1.10
	3	1.44	1.42
	4	1.21	1.26
	Average	1.21	1.26
D (45° to RD)	1	1.54	1.45
	2	1.59	1.49
	3	1.52	1.41
	4	1.32	1.29
	Average	1.49	1.41
T (90° to RD)	1	1.94	1.93
	2	1.85	1.82
	3	1.78	1.78
	4	1.32	1.49
	Average	1.72	1.76

(a) RD, sheet rolling direction

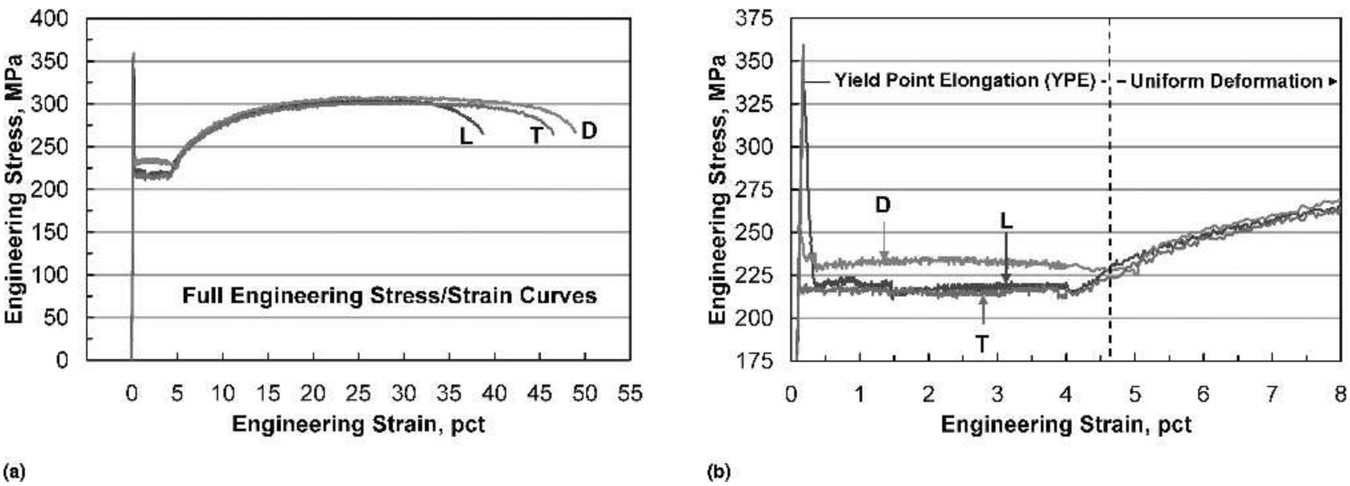


Fig. 1 Example engineering stress/strain curves for longitudinal (L), diagonal (D), and transverse (T) direction tensile tests: (a) full stress/strain curves and (b) magnified at yielding

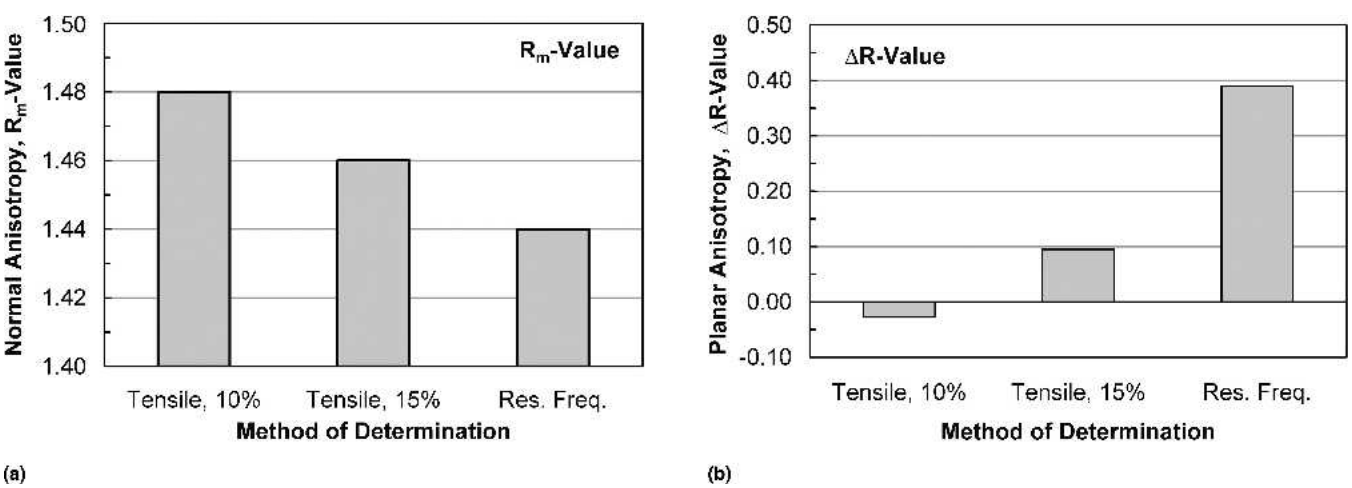


Fig. 2 The influence of measurement technique on normal and planar anisotropy parameters: (a) R_m-value and (b) ΔR-value

Excellent agreement is shown for the R_m-values, although perhaps fortuitously. For example, considering the range of R_{10} values shown in Table 4, the calculated R_m-value could have been anywhere from 1.24 to 1.64, had only one test been run in each orientation.

Poor agreement was found for the ΔR-values, where the ΔR-value ranged from slightly negative (–0.03 using R_{10} values) to significantly positive (+0.39 using the Modul-R method). This represents a significant discrepancy. For example, considering deep drawing, the ΔR-value based upon R_{10} values would suggest little or no earing in cup or can forming, and the ΔR-value determined by the Modul-R method predicts significant ear formation in the L and T directions (Ref 6). Furthermore, considering the range of R_{10} values shown in Table 4, the calculated ΔR-value could have been anywhere from –0.44 to +0.37 had only one test been run in each orientation.

3.3 Strain Dependence of R-Value during Tensile Testing

At very low strains (i.e., less than a few tenths of one percent), the concept of R-value lacks relevance for various practical and technical reasons. First, the majority of the measured deformation in this strain interval is elastic, and the volume constancy assumption (and hence Eq 2) is invalid. Secondly, minor variations in extensometer readings may translate into extremely magnified fluctuations in the R-value, as, in this strain interval, the R-value is determined by the ratio of very small numbers. Also, according to Eq 1, the R-value is, by definition, undefined at $\varepsilon_T = 0$. Nonetheless, R-values are typically calculated at strains of 10% or more, and these low-strain effects are usually insignificant contributions to the total measured strains.

Table 5 Mean normal anisotropy (R_m) and planar anisotropy (ΔR)

Anisotropy parameter	Measurement technique: Tensile, 10%	Tensile, 15%	Resonant frequency
R_m	1.48	1.46	1.44
ΔR	–0.03	0.10	0.39

For the AKDQ steel in this analysis, example R-versus-ε_L curves are shown in Fig. 3 for test numbers L-2 and D-2 from Table 4. In both examples, the R-value shows erratic behavior through the YPE portion of the stress/strain curve (up to about $\varepsilon_L = 0.05$). Similar erraticism is shown by the orthogonal strains (ε_W and ε_T), also in Fig. 3. Beyond the YPE, the curves assume more stable character, where the magnitudes of the orthogonal strains increase linearly with the length strain, and the R-value changes monotonically. The examples in Fig. 3 were chosen to illustrate two basic behavior types: (a) monotonically increasing R-value beyond YPE (Fig. 3a) monotonically decreasing R-value beyond YPE (Fig. 3b). Note that, for the Type 1 example, the orthogonal strain curves are spaced closely beyond YPE and intersect at about $\varepsilon_L = 0.1$ ($R = 1$). In the Type 2 example, the orthogonal strains are more widely spaced beyond YPE and do not intersect. Figure 4 shows R-value as a function of length strain for each set of four tensile tests in each orientation. The YPE portions of the curves and the orthogonal strains have been omitted for clarity. Note that, for each orientation, the strain dependent R-values of multiple tests tend to converge as strain increases. It appears that the aforementioned low-strain effects are not negligible when the YPE is a significant fraction of the R-value calculation strain (i.e., X in R_X), and that the nonuniform nature of discontinuous yielding greatly affects R-value calculations.

4. Discussion

To understand the influence of discontinuous yielding on R-value measurements, consideration of the incremental R-value, R′, as defined in Eq 4, is required. As an example, the data from Fig. 3 are reproduced in Fig. 5 in the strain interval from 10% to 15% elongation,a range of stable, uniform deformation. Note the linearity of the orthogonal strain plots for both examples in Fig. 5. Also note that the line fit equations for ε_W versus ε_L and ε_T versus ε_L do not pass through the origin (i.e., b in $y = mx + b$ is nonzero). Recalling the definition of λ' in Eq 4:

$$\lambda' = d\varepsilon_W/d\varepsilon_L \rightarrow d\varepsilon_W = \lambda' d\varepsilon_L \xrightarrow{\text{integrate}} \varepsilon_W = \lambda'\varepsilon_L + K \qquad \text{(Eq 6)}$$

where K is an effective offset value equal to the y-intercept of the line-fit equation for the width strain ε_W. In Fig. 5(a) (test

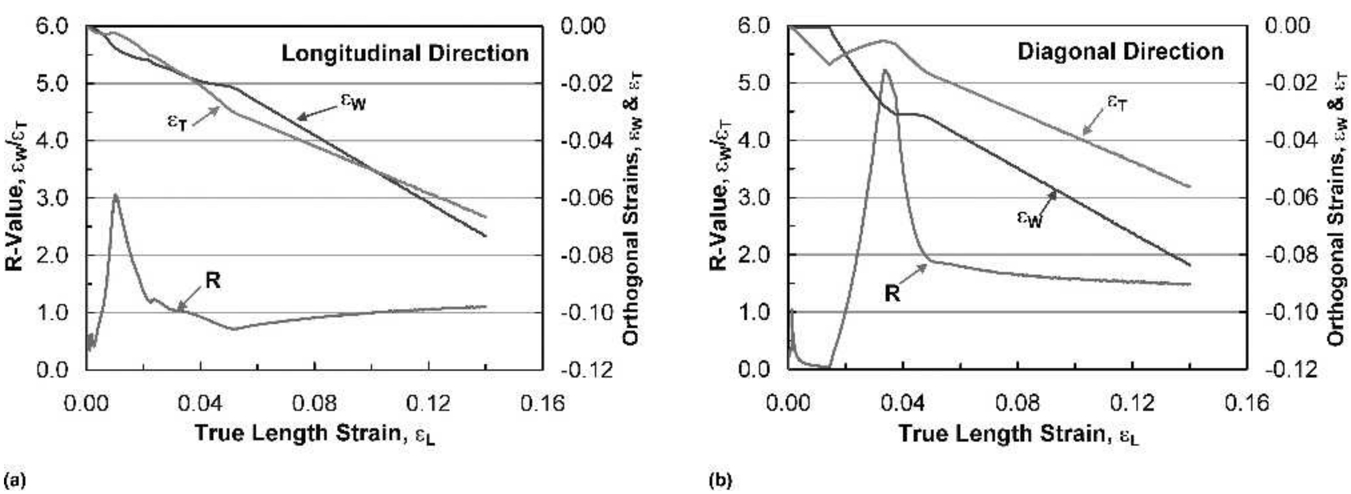
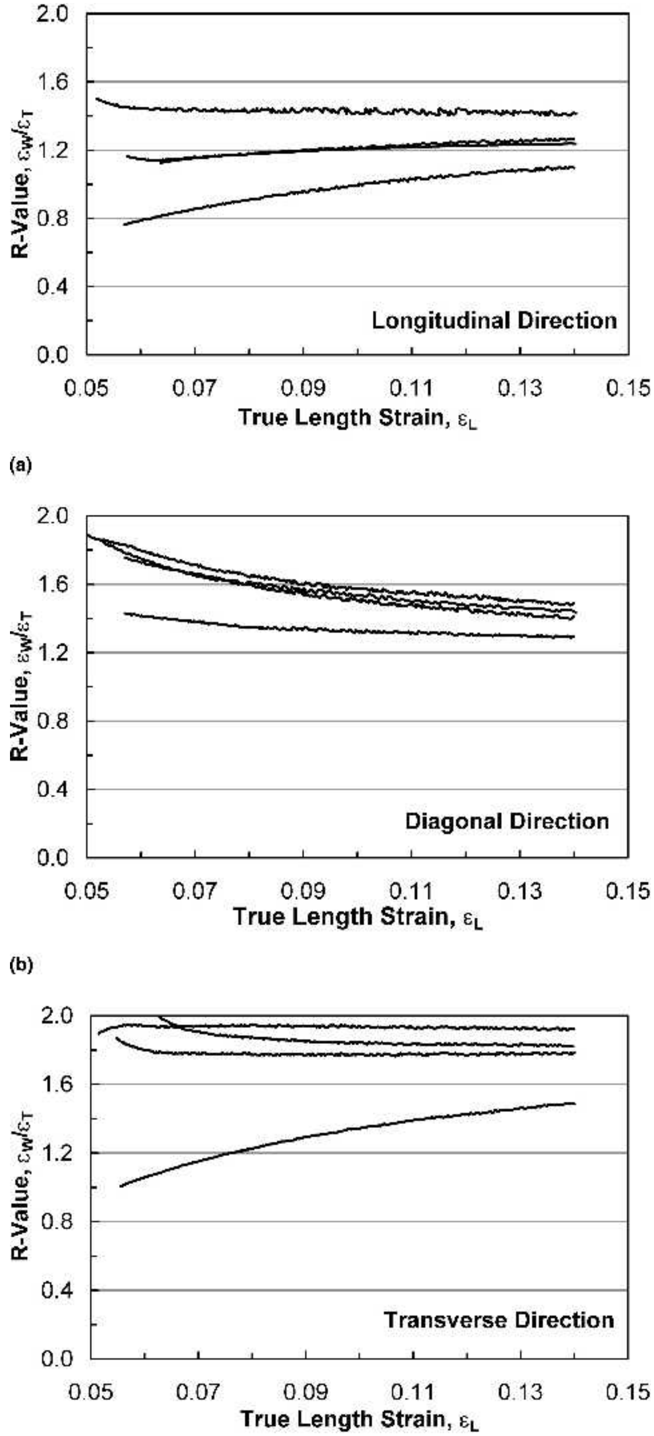


Fig. 3 Examples of R-value variation with length strain in a tensile test up to 15% elongation: (a) longitudinal direction and (b) diagonal direction. In each case, the orthogonal strains, ε_W and ε_T, are shown for reference. See text for details.



(c)

Fig. 4 *R*-value as a function of length strain for four separate tensile test repetitions in each of three orientations: (a) longitudinal direction, (b) diagonal direction, and (c) transverse direction. The YPE portions of the *R* versus ε_L curves have been omitted for clarity. Note that, for each orientation, the strain dependent *R*-values tend to converge as strain increases.

L-2), the *K*-value is positive (+0.0088), while that in Fig. 5(b) (test D-2) is negative (−0.0049). The average incremental *R*-values (R'-values) are also shown in Fig. 5 and are determined by setting λ' equal to the slope of the line fit equation for the width strain ε_W and using Eq 4. For the case of positive K-value (Fig. 5a), $R' > R$, while in the case of negative *K*-value (Fig. 5b), $R' < R$. In both cases, the *R*-value appears to approach R' asymptotically as strain increases.

In perspective, the effect of discontinuous yielding is to impose an offset to the width strain ε_W that factors into and distorts the *R*-value calculation when total plastic strains are used (Eq 1). The impact of *K* on the calculated *R*-value can be shown by substituting Eq 6 into Eq 2, such that:

$$R = \frac{\lambda' \varepsilon_L + K}{-(\varepsilon_L + \lambda' \varepsilon_L + K)} \qquad \text{(Eq 7)}$$

Figure 6 illustrates schematically the influence of *R*-value calculation strain (i.e., X in R_X) and effective offset, *K* on the *R*-value. In Fig. 6(a), the effects of positive and negative *K*-values on the measured *R*-value as a function of strain are shown, and in Fig. 6(b), the effects of *K*-value on the *R*-values measured at 10 and 15% elongation are shown (R_{10} and R_{15}, respectively). For $K = 0$ (no offset), the ε_L term factors out of Eq 7 such that $R = R'$ (Eq 4 and 7 are equivalent).

The R'-value from 10 to 15% elongation ($R'_{10\text{-}15}$) was determined for each of the twelve tests listed in Table IV (four tests each in the L, D, and T direction), and the results are summarized in Fig. 7. In Fig. 7(a), the average R'-values in each orientation are shown in contrast to the average R_{10} and R_{15} values. For the L and T directions (0° and 90° to the rolling direction), the average R'-value is greater than R_{10} and R_{15}, while in the D direction (45° to the rolling direction), R' is less than R_{10} and R_{15}. In each case, R_{15} is closer to R' than is R_{10}, as *R* approaches R' asymptotically. For further illustration, Fig. 7(b) shows the range (max-min) of each value for each group of four tests in each orientation. Clearly, the scatter in R'-values is far less than that of the *R*-values, and there is more variability in R_{10} than in R_{15}. When the average R'-values are used to determine R_m and ΔR (substitute R'_0, R'_{45}, and R'_{90} for R_0, R_{45}, and R_{90} in Eq 5), $R'_m = 1.42$ and $\Delta R' = 0.39$, nearly identical to the values predicted by the resonant frequency (Modul-R) method (Table 5). It is anticipated that, if the subject AKDQ steel were temper rolled to remove the YPE, the R_{10} and R_{15}-values would be closer to, if not equal to, the R'-values determined in this analysis.

4.1 Implications

The relationship between the amount of YPE and the effective offset *K* has not been established, nor should it be inferred from the limited data presented herein. The mechanics of Lüders band formation and propagation is complex and beyond the scope of this analysis (Ref 12). It seems rather fair to project, however, that the erratic behavior illustrated in this analysis is a reflection of the heterogeneous and variable nature of discontinuous yielding, and that larger amounts of YPE will cause larger variations in anisotropy parameters. For adequately temper-rolled steels and for inherently YPE-free materials (e.g., fully stabilized steels, dual phase steels, and certain Al alloys), the *R*-value can be determined in the conventional way (Eq 1) with confidence, as long as the mechanical testing procedures are sound. Using R' in place of *R* is generally a good practice in cases where precise measurements are needed for sheet metal forming simulation purposes—not only in the case of significant YPE, but also in cases where actual strain-dependent plastic anisotropy is expected (i.e., unrelated to testing aberrations or discontinuous yielding).

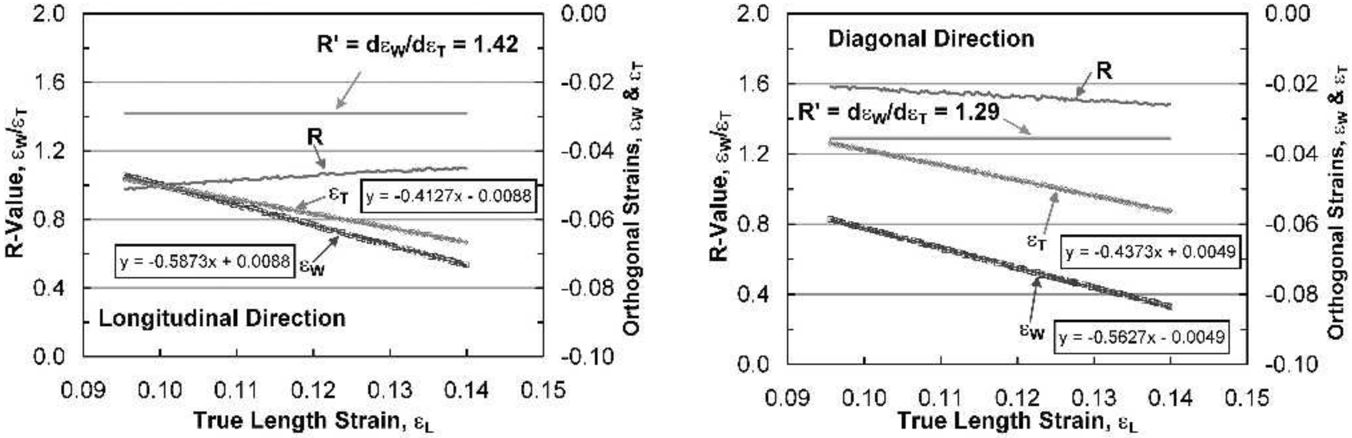


Fig. 5 *R*-value as a function of length strain within a region of stable, uniform deformation from 10 to 15% elongation: (a) longitudinal direction and (b) diagonal direction. The data represent the same examples shown in Fig. 3. Note the linearity of the orthogonal strain plots (ε_W versus ε_L and ε_T versus ε_L) within this strain interval. The average incremental *R*-value, $R' = d\varepsilon_W/d\varepsilon_T$ is shown for contrast with the conventional *R*-value, $R = \varepsilon_W/\varepsilon_T$.

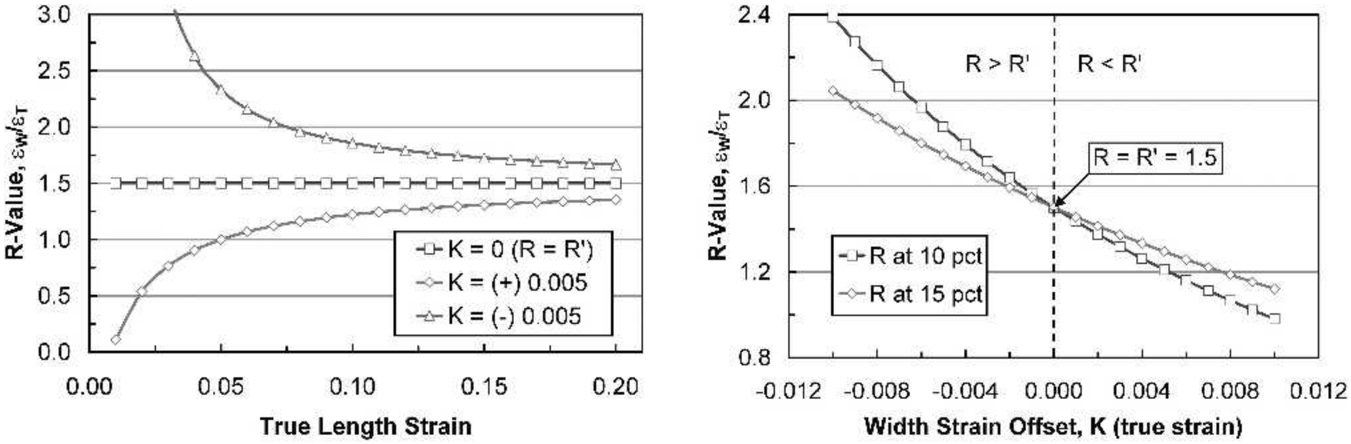


Fig. 6 Schematic illustration of the effects of *R*-value calculation strain (i.e., X in R_X) and effective offset *K* on the *R*-value. In this example, the average incremental *R*-value, R' is 1.5: (a) effects of positive and negative *K*-values on the measured *R*-value as a function of strain and (b) effects of *K*-value on the *R*-values measured at 10 and 15% elongation (R_{10} and R_{15}, respectively).

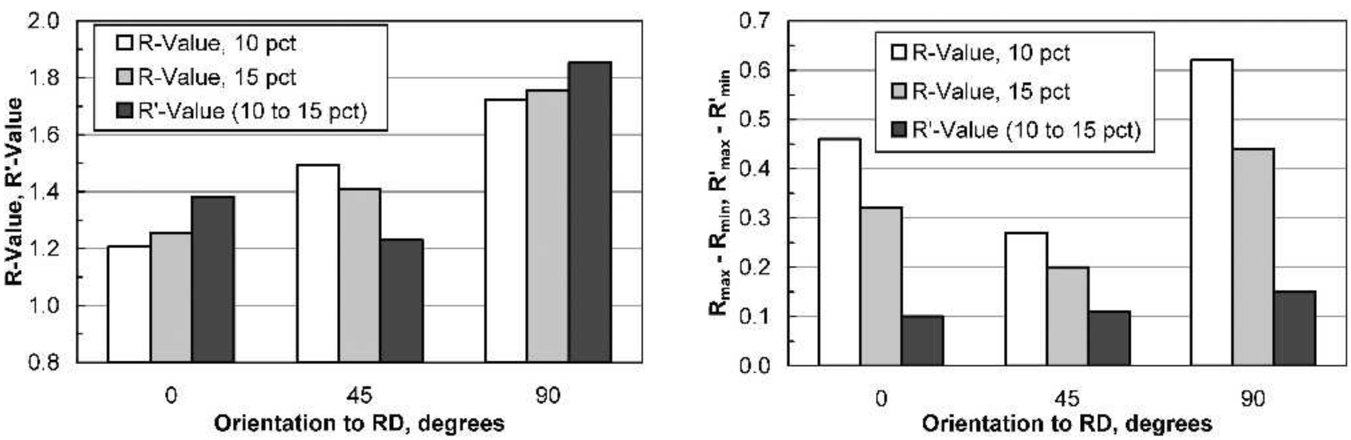


Fig. 7 *R*-values and R'-values in various orientations with respect to the sheet rolling direction (RD): longitudinal (0°), diagonal (45°), and transverse (90°). The conventional *R*-values are shown at 10 and 15% elongation (R_{10} and R_{15}, respectively), and the R'-values are determined in the 10-15% elongation strain interval. (a) Average values of each parameter in each orientation; (b) the range (max-min) of each parameter for four tests in each orientation

5. Conclusions

In this study, the influence of discontinuous yielding on normal anisotropy measurements was examined for a laboratory-annealed, nontemper-rolled AKDQ sheet steel. The following conclusions were drawn from the results of the analysis.

- Discontinuous yielding (yield point elongation or YPE) significantly influences *R*-value calculations by imposing an offset to the width strain measurement during a sheet tensile test, if the *R*-value is defined in terms of total strains ($R = \varepsilon_W/\varepsilon_T$).
- The width strain offset (*K*-value) may be positive or negative and shows considerable variability between tests. Negative and positive *K* values lead to artificially high and artificially low measured *R*-values, respectively.
- The incremental *R*-value ($R' = d\varepsilon_W/d\varepsilon_T$) is a more accurate and consistent representation of normal anisotropy and should be used in cases of significant yield point elongation (YPE), or in any case where actual strain-dependent plastic anisotropy is expected.
- In the case of a nonzero *K*-value, the measured *R*-value approaches the R'-value asymptotically as strain increases and as the percent YPE becomes a lesser fraction of the total measured strains in a tensile test.
- For adequately temper rolled steels, and for inherently YPE-free materials, the *R*-value can be determined in the conventional way, based upon total plastic strains.

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